

TTP 95–22
Technion-PH-95-3
May 1995
hep-ph/9505288

PARTON HADRON DUALITY IN NONLEPTONIC B HADRON DECAYS

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Abstract

Assuming so called global duality we argue that it is very likely that local duality needed to obtain results for the hadronic width of heavy meson decays within the $1/m_Q$ expansion holds. Hence, if the discrepancy between experiment and the theory concerning charm counting, the semileptonic branching fraction and the lifetimes of b hadrons persist, it may be taken as a hint at some qualitatively new effect in (nonperturbative) QCD or even as a new physics.

1 Introduction

The $1/m_Q$ expansion allows us to perform calculations of inclusive heavy hadron decay rates in a model independent and QCD based framework [1]. Unlike for exclusive decays we are even able to deal with purely hadronic processes using this method based on operator product expansion and Heavy Quark Effective Theory. This in turn allows us to calculate lifetimes and branching fraction for b hadrons within this QCD based approach.

The leading order of the $1/m_Q$ expansion is generically simply the parton model expression. The corresponding decay rates in the parton model are by now known for more than twenty years, and two problems with the rates calculated in this way have been noticed since then. Firstly, in the pure parton model, the semileptonic branching fraction comes out to be too large as compared to the measurements and secondly in the parton model all b hadron lifetimes are the same and equal the b quark lifetime.

The hope was to fix these problems by including perturbative and nonperturbative corrections originating from terms of order $\alpha_s(m_Q)$ and $1/m_Q^2, 1/m_Q^3$. It turned out soon that the nonperturbative contributions are way too small to explain the semileptonic branching fraction [2] as well as the lifetime difference between the B mesons and the Λ_b baryon [3]. In fact the nonperturbative contributions are so small that they can safely be ignored given the present experimental situation.

The purely perturbative corrections to the semileptonic decays have been calculated some time ago [5] and were also found to be too small to explain the data. The perturbative corrections to the hadronic decays have also been calculated in the late seventies for the case of vanishing masses of the final state quarks; again they are not sufficient to explain the semileptonic branching fraction (see e.g. refs. [4, 2] for the recent reviews).

This was the motivation for Bagan and coworkers [6] to perform the full $\mathcal{O}(\alpha_s)$ calculation including the mass effects of the final state c quarks in the channel $b \rightarrow c\bar{c}s$. In fact, Bagan et al. find a substantial enhancement for this channel, which, together with the more modest enhancement in $b \rightarrow \bar{c}ud$ channel due to the same mechanism almost explains the semileptonic branching fraction. This channel has also been discussed previously in this context in [7], where it was argued that the operator product expansion could fail due to the small energy release in this channel, since the c quarks are heavy, or that parton hadron duality could fail due to resonances that are close to the point where the hadronic spectral function is replaced by the partonic one.

However, one cannot blame the problem with the semileptonic branching fraction completely to the $b \rightarrow c\bar{c}s$ channel, since this would increase this channel by something like (30 - 40)%. This on the other hand contradicts charm counting in B decays, since this would give for the number n_c of c and \bar{c} quarks created per B decay $n_c \sim 1.3$ which is about two standard deviations away from the experi-

mentally observed value of $n_c = 1.07 \pm 0.07$ [9]. The QCD radiative corrections calculated in [6] also enhance this channel; the full $\mathcal{O}(\alpha_s)$ calculation also gives $n_c \sim 1.3$, again in disagreement to charm counting.

Another problem for the $1/m_Q$ expansion is the lifetime ratio of the Λ_b baryon and the B meson. While the $1/m_Q$ expansion predicts a value in excess of 0.9 for the ratio $\tau(\Lambda_b)/\tau(B^+)$, recent measurements yield $\tau(\Lambda_b)/\tau(B^+) = 0.7 \pm 0.1$ [10] which is a deviation at the level of two standard deviations.

These two problems, namely the semileptonic branching fraction in combination with charm counting and the lifetime of the Λ_b baryon, have attracted considerable attention over the last time, although they are not yet of convincing significance. Neither the nonperturbative nor the perturbative corrections seem to be big enough to explain the problems [2]. In particular, as it was mentioned above, although the recently completed calculation of the complete $\mathcal{O}(\alpha_s)$ correction [6] is close to explain the discrepancy in the semileptonic branching fraction, it does it by the price of generating disagreement with charm counting.

All the arguments used in the $1/m_Q$ expansion are based on parton-hadron duality and it has been suggested that a breakdown of duality is the explanation of these discrepancies. It has been pointed out that unlike in the calculation of the semileptonic branching fraction, where *global* duality is needed, in the calculation of the hadronic width of the B meson *local* duality needs to be invoked: In the calculation of the semileptonic width only a weighted average of the spectral function of hadronic operators is replaced by the corresponding average of the parton model expression, while for the hadronic width one replaces the hadronic spectral function at a fixed kinematic point (i.e. locally) by the partonic expression.

In this short note we shall assume that global duality holds and investigate to what extent the hadronic spectral function may deviate from the parton model expressions. Our arguments are necessarily qualitative, since it involves the non-perturbative sector of QCD. Intuitively one expects that the final state quarks may form resonance states which may lead to a hadronic spectral function different from the partonic one. We shall use simple resonance models to study the possible effects.

2 Parton Hadron Duality

We shall start from the effective Hamiltonian describing nonleptonic B decays, neglecting $b \rightarrow u$ transitions and penguins

$$H_{eff} = \frac{G_F}{\sqrt{2}} V_{cb} \sum_{U=u,c} \left[C_2 (\bar{b} \gamma_\mu (1 - \gamma_5) c) (\bar{U} \gamma_\mu (1 - \gamma_5) D_U) \right. \\ \left. + C_1 (\bar{b} \gamma_\mu (1 - \gamma_5) D_U) (\bar{U} \gamma_\mu (1 - \gamma_5) c) \right] \quad (1)$$

where D is the Cabbibo-rotated down type quark field

$$\begin{aligned} D_u &= V_{ud}^* d + V_{us}^* s \\ D_c &= V_{cd}^* d + V_{cs}^* s \end{aligned} \quad (2)$$

and C_1 and C_2 are the Wilson coefficients in the leading log approximation

$$\begin{aligned} C_1 &= \frac{1}{2} \left[\left(\frac{\alpha_s(M_W)}{\alpha_s(m_b)} \right)^{\gamma_+} - \left(\frac{\alpha_s(M_W)}{\alpha_s(m_b)} \right)^{\gamma_-} \right] \\ C_2 &= \frac{1}{2} \left[\left(\frac{\alpha_s(M_W)}{\alpha_s(m_b)} \right)^{\gamma_+} + \left(\frac{\alpha_s(M_W)}{\alpha_s(m_b)} \right)^{\gamma_-} \right] \end{aligned} \quad (3)$$

Here

$$\gamma_+ = \frac{6}{33 - 2N_f} = -\frac{1}{2}\gamma_-.$$

Using this Hamiltonian we may define the forward matrix element

$$T(q^2, vq) = \int d^4x e^{-iqx} \langle B(v) | H_{eff}(x) H_{eff}(0) | B(v) \rangle \quad (4)$$

where $v = P_B/m_B$ is the velocity of the B meson.

We shall use the correlator T to study parton hadron duality. The analytical structure of T for complex vq and fixed q^2 is given by the intermediate states $|X\rangle$ that may be excited by H_{eff}

$$T(q^2, vq) = \int d^4x (2\pi)^4 \delta^4(m_B v - q - P_X) |\langle B(v) | H_{eff} | X \rangle|^2. \quad (5)$$

The support of the function T may easily be calculated. For a fixed q^2 , T is nonvanishing in the interval

$$vq < \frac{1}{2m_B}(q^2 + m_B^2 - \mu^2) \text{ and } vq > \frac{1}{2m_B}(q^2 + 2m_B^2 + \mu^2) \quad (6)$$

where μ is the mass of the lightest hadronic state which may appear in the sum. In our case this state is a $D\pi$ state where both the D and the π are at rest, so $\mu = m_D + m_\pi$.

In the parton model these intermediate states are three quark states at the tree level, and the function T may be calculated to be

$$\begin{aligned} T(q^2, vq) &= \left(\frac{G_F^2 m_b^4}{96\pi^3} \right) \langle B(v) | \bar{b}b | B(v) \rangle N_c \left(C_1^2 + C_2^2 + \frac{2C_1 C_2}{N_c} \right) V_{cb} \\ &\quad \left[V_{ud}^* F_2(m_c^2/Q^2) + V_{cs}^* F_3(m_c^2/Q^2) \right] \end{aligned} \quad (7)$$

where $Q = m_B v - q$. Furthermore, we have neglected CKM suppressed terms proportional to V_{us} and V_{cd} and

$$F_2(x) = (1 - x^2)(1 - 8x + x^2) - 12x^2 \ln(x) \Theta(1 - x) \quad (8)$$

$$F_3(x) = v(1 - 14x - 2x^2 - 12x^3) + 24x^2(1 - x^2) \ln \frac{1+v}{1-v} \Theta(1 - 4x), \quad (9)$$

$$v = \sqrt{1 - 4x} \quad (10)$$

Note that the functions F_2 and F_3 have support in the intervals

$$vq < \frac{1}{2m_B}(q^2 + m_B^2 - m_c^2). \quad (11)$$

and

$$vq < \frac{1}{2m_B}(q^2 + m_B^2 - 4m_c^2) \quad (12)$$

respectively.

Parton hadron duality means that the correlator T_{parton} calculated in the parton model is “on the average” equal to the true hadronic correlator T_{hadron}

$$\int d(vq) T_{parton}(q^2, vq) w(vq) = \int d(vq) T_{hadron}(q^2, vq) w(vq), \quad q^2 \text{ fixed} \quad (13)$$

where w is some smooth weighting function having support over some averaging interval. Close to the regions where only the $D\pi$ or $J/\Psi K$ states contribute the partonic and the hadronic results have in general a quite different shape, and the interval over which one needs to average in (13) is larger than if we consider the two functions at higher invariant masses. This situation has been analyzed for the case of the total cross section $e^+e^- \rightarrow \text{hadrons}$ in [11].

For the calculation of the total hadronic width we have to consider the correlator at the point where the invariant mass of the final state hadrons is equal to the B meson mass

$$\Gamma(B \rightarrow \text{hadrons}) \propto T(0, 0), \quad (14)$$

and the common folklore is that at this high invariant mass one may safely shrink the interval of averaging in (13) to a point and simply equate the partonic with the hadronic correlator. In other words, we shall assume that “global” duality in the sense of a large averaging interval holds and discuss under which circumstances we may shrink the averaging interval to a single point, thereby going from “global” to “local” duality.

The situation in the nonleptonic decays has to be compared with the one in semileptonic decays [12, 13], where the hadronic contribution is the product of the two hadronic currents. In this case q may be identified with the momentum transferred to the leptons. While the analytical structure is quite similar (the lowest state being a D meson at rest) the integration over the neutrino momentum introduces a smooth averaging such that parton hadron duality is guaranteed

in almost all phase space for the charged lepton, except for the endpoint, where the energy of the charged lepton becomes maximal. Only if we would consider doubly differential distributions such that the invariant mass of the final hadronic state would be fixed, a similar situation would occur as in the nonleptonic decays, namely that the interval over which we average shrinks to a point. In this sense one expects that the calculation of the lepton spectrum as well as of the total semileptonic rate is more reliable than the calculation of the total nonleptonic rate, since in the semileptonic case only global duality is needed, while the nonleptonic case rests on the assumption of global duality.

3 Resonance Model

Motivated by this reasoning we have considered the correlator for the nonleptonic decays in the context of very simple model. The idea is to identify possible contributions which could generate some structure of the hadronic result which could result in a difference between $T_{parton}(0,0)$ and $T_{hadron}(0,0)$ hence a violation of local duality, although global duality holds. As pointed out above, the partonic tree level result is a smooth function of $Q^2 = m_b^2 + q^2 - 2m_b(vq)$, since it is obtained from a three particle phase space for the three final state quarks. One possibility one may consider is that two of these final state quarks form a resonance, corresponding e.g. to the semiinclusive final processes $B \rightarrow D_{(s)} + \text{anything}$ or $B \rightarrow J/\Psi + \text{anything}$. In order to study the qualitative features of this idea, we consider as a simple model for such a situation the effective Hamiltonian

$$H'_{eff} = \frac{G_F}{\sqrt{2}} f_\phi (\bar{b} \gamma_\mu (1 - \gamma_5) q) \phi^\mu \quad (15)$$

where ϕ_μ is the field of a vector like resonance of mass M made of a quark-antiquark pair, and f_ϕ is its coupling strength to the left handed color singlet $b \rightarrow q$ current, including all the CKM matrix elements. A similar model has been introduced by Stech and Palmer [14] to analyze semiinclusive nonleptonic B decays.

It is a simple exercise to calculate the corresponding correlator. The resonance contribution to T near threshold turns out to be

$$T(q^2, vq) = \frac{G_F^2}{16\pi} f_\phi^2 \langle B(v) | \bar{b} b | B(v) \rangle \left(\frac{1}{Q^2} \right)^2 \quad (16)$$

$$\sqrt{(Q^2 - (M - m_q)^2)(Q^2 - (M + m_q)^2)} \Theta(Q^2 - (M - m_q)^2)$$

$$\left[Q^2 - M^2 + m_q^2 + \frac{1}{M^2} ((Q^2 - m_q^2)^2 - M^4) \right]$$

Contributions of other resonances to the spectral function are smooth since these resonances lie away from threshold, and we do not write it explicitly. Generically

the phase space contributes the factor involving the square root, and in the limit in which $M, Q^2 \gg m_q$ the r.h.s. of (16) is a smooth function even at threshold

$$T(q^2, vq) = \frac{G_F^2}{16\pi} f_\phi^2 \langle B(v) | \bar{b}b | B(v) \rangle \left(\frac{1}{Q^2} \right)^2 \frac{1}{(Q^2 - M^2)^2 (Q^2 + M^2) \Theta(Q^2 - M^2)} \quad (17)$$

In our case m_q is just the mass of the light strange quark. This means that a scenario of a resonance as modelled in (15) such that the threshold is close to the mass of the B meson does not introduce any rapid variations in the hadronic spectral function. Furthermore, since we assume global duality in the sense of (13), one finds that the hadronic and partonic spectral functions cannot be drastically different, at least not in a scenario as modelled by the Hamiltonian (15). Hence we conclude that it is quite likely that the partonic correlator T is a good approximation to the hadronic, even for the case of a resonance as we introduced above fairly close to the mass of the B meson.

Typically the correlators obtained from intermediate states with two or more partons or hadrons are smooth functions of the variable $Q^2 = m_B^2 + q^2 - 2m_b(vq)$ and any resonance formation involving at least a two particle intermediate state does not introduce rapid variations of the correlator. For all these states we may apply the same argument as above, leading to the statement that it is quite likely that local duality holds. In this framework, the only way to introduce strong variations is if there is a relatively narrow resonance R_c leading to a single particle intermediate state in the correlator T .

In order not to get into conflict with charm counting, this resonance state has to appear in the channel $b \rightarrow c\bar{u}d$, thereby enhancing this channel by some amount, which then would explain the semileptonic branching fraction without enhancing the charm created in B decays. In this case, the correlator would take the form

$$T(q^2, vq) = 2M\Gamma \frac{\langle B(v) | H_{eff} | R_c \rangle \langle R_c | H_{eff} | B(v) \rangle}{(Q^2 - M^2)^2 + M^2\Gamma^2} \quad (18)$$

where M and Γ are the mass and the width of R_c respectively. If this resonance would be sufficiently narrow and its mass sufficiently close to the mass of the B meson, parton hadron duality could still hold on the average, but the interval of averaging would simply be too large to justify the calculation of the total hadronic width using local duality.

However, this possibility is quite unlikely. First of all, it would not help to explain the lifetime problem with the Λ_b baryon, since such a resonance state would increase the width of the B mesons due to an increase in the channel $b \rightarrow c\bar{u}d$, and hence would make their lifetime shorter compared to the parton model value. Assuming that there is not such a resonance in the channel involving the baryons, this would increase the ratio $\tau(\Lambda_b)/\tau(B)$, which will make the discrepancy even stronger.

Furthermore, the effect of such a resonance would be visible as well in the semileptonic channel, if the resonance state would have isospin $I = 1/2$. However, one may avoid any influence on the semileptonic decays, if the isospin of R_c is $I = 3/2$. This would mean that in a quark model picture this state is a four quark state consisting of a two quarks and two antiquarks.

However, the $b \rightarrow c\bar{u}d$ piece of the effective Hamiltonian (1) is an isospin triplet with $I_3 = -1$, and hence the hadronic decays of the charged B mesons will have only final states with isospin $I = 3/2$, while the final states of the nonleptonic decays of the neutral B meson will be a mixture of $I = 3/2$ and $I = 1/2$. Consequently an enhancement of the $I = 3/2$ contribution through a resonance would make the lifetime of the charged B meson shorter than the one of the neutral states, which is in conflict with the data on the lifetime ratio between charged and neutral B mesons, which indicate that the lifetimes are equal $\tau(B^+)/\tau(B^0) = 1.009 \pm 0.069$ [9]. Hence there is not much room for such a resonance state and even if it would exist, its contribution would be not sufficient to explain the semileptonic branching fraction problem.

4 Conclusions

Even after more detailed calculations of the QCD radiative corrections the situation concerning charm counting and the semileptonic branching fraction remains inconclusive. A failure of local parton hadron duality as it is needed to obtain results within the $1/m_Q$ expansion has been considered as a possible solution of the problem, but we have argued based on models that local parton hadron duality likely to hold, if global duality is assumed.

Global duality, on the other hand, is a concept without which we could not relate many theoretical results to data, which in turn are compatible with the assumption of global duality. Hence the semileptonic branching fraction problem and the charm counting remain open questions in view of the present data. Either the data on charm counting change, such that $n_c \sim 1.3$. In this case the problem is solved, and such a result would also be in agreement with the recent calculation of the full $\mathcal{O}(\alpha_s)$ corrections [6]. However, if data remain as they are and the errors become smaller, than there have to be either yet unknown effects in nonperturbative QCD, or the discrepancies find an explanation in terms of physics beyond the standard model [15, 16]. However, we find an explanation in terms of QCD effects more likely, since physics beyond the standard model involves scales much larger than the B meson mass and hence will affect the decays of the b quark rather than the ones of the hadrons, in other words, such an explanation cannot be the source of the Λ_b lifetime problem.

Acknowledgements

The authors thank M. Gronau and M. Beneke and especially M. Shifman and A. Vainshtein for very usefull discussions. T.M. acknowledges the hospitality of the particle theory group of the Technion, where this work has been initiated. The research of B.B. was supported by the Israel Academy of Sciences under the contract 938041 and by the Technion V.P.R. fund.

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